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The equation of state of matter at ultra-high densities

Abstract. In this work it is shown that Zeldovich's results for the behaviour of matter at ultra-high densities are not valid when many-body forces are included.

There has been much interest in ultra-high density matter with respect to subjects such as cosmology, gravitational collapse and neutron stars. The equation of state for such matter is usually calculated assuming that the system consists of particles interacting via a potential. The validity of this assumption is doubtful but can hardly be avoided until better techniques are available.

The speed of sound D, as calculated for most such equations of state for ultra-high density matter, increases with density and eventually exceeds that of light. Such equations of state are usually cut off (Tsuruta and Cameron 1966) at the density when $D = c$.

Zeldovich (1962) has constructed a model in which the limit $D = c$ is reached as the number density $n \to \infty$. He considers a system of a large number of baryons (mass M) interacting via the exchange of vector mesons of non-zero mass μ. The interaction between two static baryons is shown to be repulsive and of the form

$$V = \frac{g^2 \exp(-\mu r_{12})}{r_{12}}$$

where g is a constant. He then shows that the energy per unit volume is

$$\frac{E}{\Omega} = Mn + \frac{2\pi g^2 n^2}{\mu^2}$$

so that the pressure $p = 2\pi g^2 n^2/\mu^2$. Hence $p \to E/\Omega$ and $D \to c$ as $n \to \infty$.

By arguing that the potential V may be density dependent, because of the exchange of a number of mesons, Harrison (1965) has obtained a modification of Zeldovich's result so that $p \to \frac{1}{3}E/\Omega$ as $n \to \infty$.

In Zeldovich's model (and also in Harrison's model) of a system of baryons interacting via the exchange of mesons, it has been tacitly assumed that the static potential energy $V(\mathbf{r}_1, \mathbf{r}_2, \ldots, \mathbf{r}_N)$ of N fixed baryons can be expressed as a sum of two-body potentials $\Sigma_{i \neq j} V_2(\mathbf{r}_i, \mathbf{r}_j)$. However, it has been known for a long time (Primakoff and Holstein 1939) that the potential energy of a system of particles cannot be expressed as the sum of two-body potentials and is instead of the form

$$V(\mathbf{r}_1, \mathbf{r}_2, \ldots, \mathbf{r}_N) = \sum_{i \neq j} V_2(\mathbf{r}_i, \mathbf{r}_j) + \sum_{i \neq j \neq k} V_3(\mathbf{r}_i, \mathbf{r}_j, \mathbf{r}_k) + \ldots + V_N(\mathbf{r}_1, \mathbf{r}_2, \ldots, \mathbf{r}_N)$$

where V_j is a j-body potential. Usually (particularly at low densities) the many-body potentials $V_j (j \geqslant 3)$ are negligible. In this work we shall show that at high densities many-body potentials are the ones which become dominant, so that Zeldovich's results are no longer valid. As in Zeldovich's model we shall neglect effects such as particle creation, velocity dependence of potential etc.

We consider a system of baryons (fermions), interacting via many-body potentials which we assume to depend only on the interparticle distances. Thus the Hamiltonian for the system is

$$H = \int \mathrm{d}\mathbf{r}_1 \, \psi^\dagger(1) \frac{\nabla_1{}^2}{2M} \psi(1) + \sum_{j=2}^{N} \frac{1}{j!} \int \mathrm{d}\mathbf{r}_1 \, \mathrm{d}\mathbf{r}_2 \ldots \mathrm{d}\mathbf{r}_j \times \psi^\dagger(1) \psi^\dagger(2) \ldots \psi^\dagger(j) \, V_j(\mathbf{r}_1, \ldots, \mathbf{r}_j) \, \psi(j) \ldots \psi(1). \quad (1)$$

Here, $\psi(1)$ stands for the annihilation operator for a fermion at $\boldsymbol{r}_1$ at time t_1, $\psi^{\dagger}(1)$ being the corresponding creation operator. In equation (1) we take $t_1 = t_2 = \ldots = t_N$.

We define the one-particle Green's function

$$G(1,1') = \frac{1}{i}\langle T(\psi(1)\psi^{\dagger}(1'))\rangle$$

where the expectation value is taken with respect to the exact ground state. Using standard methods (Kadanoff and Baym 1962) the 'equation of motion' for the one-particle Green's function is given by

$$\left(\mathrm{i}\frac{\partial}{\partial t_1} + \frac{\nabla_1{}^2}{2M}\right) G(1,1') = \delta(1-1') + \sum_{j=2}^{N} \frac{(-\mathrm{i})^{j-1}}{(j-1)!}\int \mathrm{d}\boldsymbol{r}_2\,\mathrm{d}\boldsymbol{r}_3 \ldots \mathrm{d}\boldsymbol{r}_j$$
$$\times\, G_j(12\ldots j, 1'2^+ \ldots j^+)\, V_j(\boldsymbol{r}_1, \boldsymbol{r}_2, \ldots \boldsymbol{r}_j) \tag{2}$$

where $t_1 = t_2 = \ldots = t_j$ and 2^+ stands for $\boldsymbol{r}_2$, $t_2+\delta$. In the Hartree approximation we write

$$G_j(123\ldots j, 1'2^+ \ldots j^+) \simeq G(1,1')G(2,2^+)\ldots G(j,j^+).$$

Furthermore

$$\mathrm{i}G(2,2^+) = \langle n(\boldsymbol{r}_2)\rangle.$$

For a translationally invariant system (Kadanoff and Baym 1962) we write

$$\langle n(\boldsymbol{r}_2)\rangle = n$$

(the number density) so that equation (2) becomes

$$\left\{\mathrm{i}\frac{\partial}{\partial t_1} + \frac{\nabla_1{}^2}{2M} - \sum_{j=2}^{N}\frac{(-\mathrm{i})^{j-1}}{(j-1)!}\frac{n^{j-1}}{\mathrm{i}^{j-1}}a_j\right\} G(1,1') = \delta(1-1'). \tag{3}$$

Here

$$a_j = \int \mathrm{d}\boldsymbol{r}_2 \ldots \mathrm{d}\boldsymbol{r}_j\, V_j(\boldsymbol{r}_1, \ldots, \boldsymbol{r}_j)$$

is a constant, independent of $\boldsymbol{r}_1$ for a translationally invariant system. The Fourier transform of the one-particle Green's function is hence given by

$$G(p,\omega) = \{\omega - \epsilon(p)\}^{-1}$$

where

$$\epsilon(p) = \frac{p^2}{2M} + \sum_{j=2}^{N}\frac{a_j}{(j-1)!}n^{j-1}.$$

Since $\epsilon(p_{\mathrm{F}}) = (\partial E/\partial N)_{\Omega}$ where p_{F} is the Fermi momentum and N is the total number of particles in a volume Ω, it follows that

$$\frac{E}{\Omega} = \frac{3(3\pi^2)^{2/3}}{10M}n^{5/3} + \sum_{j=2}^{\infty}\frac{a_j}{j!}n^j. \tag{4}$$

If we set $V_j = 0$ for $j > 2$ then $a_j = 0$ for $j > 2$. Further, if we take the two-body potential to be the one used by Zeldovich, $V_2 = g^2\exp(\mu r_{12})/r_{12}$, it is easy to show that $a_2 = 4\pi g^2/\mu^2$. Hence, as $n \to \infty$,

$$\frac{E}{\Omega} \sim \frac{2\pi g^2 n^2}{\mu^2}$$

which is precisely the result obtained by Zeldovich. Note that even if we had used the correct relativistic expression for the kinetic energy and included the rest-mass energy, the potential energy would have been dominant in the high-density limit.

If the many-body potentials are included in equation (4) a_j is positive if V_j is repulsive, and negative if V_j is attractive. Many-body potentials are extremely difficult to derive and in general may be repulsive or attractive. In this connection it is interesting to note that the static nuclear many-body potentials derived using old-fashioned perturbation theory (Drell and Huang 1953) were found to be repulsive for j odd and attractive for j even. Hence, as $n \to \infty$, the behaviour of E/Ω is not known, quite contrary to the results of Zeldovich and Harrison.

It should be noted that the true ground-state energy is lower than that given in the Hartree approximation because of the neglect of the Hartree–Fock and correlation energies. These corrections were also neglected by Zeldovich.

In conclusion, we find that the energy per unit volume in the Hartree approximation is given by a power series in the density. The terms in n^3, n^4, ... come from the contributions to E/Ω of the 3, 4, ... body potentials respectively and were not included by Zeldovich and Harrison. As $n \to \infty$ it is not known whether E/Ω tends to a limit or not so that we cannot say anything about the corresponding behaviour of the speed of sound. This is hardly surprising considering the complexity of ultra-dense matter. We suggest that, if all the many-body potentials are properly taken into account, the speed of sound will not exceed that of light.

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Experimental observation of the drift-dissipative instability in afterglow plasmas

Abstract. This letter presents results which support the general shape of the ω against k_z dispersion curve for the drift-dissipative instability, predicted using the 'two-fluid' equations in slab geometry. The wave is observed to be self-excited in the k_z region where a positive growth rate is predicted.

In afterglow plasmas, the usual possible causes of instability such as axial current, non-isotropic velocity distributions, imposed electric fields, etc., are absent. Consequently, most experiments on afterglows have shown them to be stable; however,

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